▸ AMíLCAR SERNADAS, *Extending classical logic for reasoning about quantum systems.*
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The exogenous quantum propositional logic (EQPL) was proposed in [13, 14, 15] for modeling and reasoning about quantum systems, embodying all that is stated in the relevant postulates of quantum physics (as presented, for instance, in [9, 16]). The logic was designed from the semantics upwards, starting with the key idea of adopting superpositions of classical models as the models of the proposed quantum logic.

The EQPL approach to quantum reasoning is quite different from the mainstream approach [8, 10]. The latter, as initially proposed by Birkhoff and von Neumann [6], focuses on the lattice of closed subspaces of a Hilbert space and replaces the classical connectives by new connectives representing the lattice-theoretic operations, while the semantics of the former uses superpositions of classical models, leading to a natural extension of the classical language containing the classical connectives (just as modal languages are extensions of the classical language). Furthermore, EQPL allows quantitative reasoning about amplitudes and probabilities, being in this respect much closer to the possible-worlds logic for probability reasoning than to the mainstream quantum logics. Finally, EQPL is designed to reason about finite collections of qubits and, therefore, it is suitable for applications in quantum computation and information. In fact, each EQPL model is a superposition of classical valuations that corresponds to a unit vector expressed in the computational basis of the Hilbert space resulting from the tensor product of the independent qubit systems.

It is possible to express in EQPL a wide range of properties of states of such a finite collection of qubits. For example, one can impose that some qubits are independent of (that is, not entangled with) other qubits; one can prescribe the amplitudes of a specific quantum state; one can assert the probability of a classical outcome after a projective measurement over the computational basis; and, one can also impose classical constraints on the admissible quantum states.

The talk is focused on a decidable fragment of EQPL obtained by relaxing the semantic structures of EQPL [7]: inner product spaces over an arbitrary real closed field and its algebraic closure are used instead of Hilbert spaces. The decidability results from the fact that the first order theory of such fields is decidable [5, 17], using a technique inspired by related work on probabilistic logic [1].

More recent developments are mentioned at the end of the talk, namely the exogenous quantum temporal propositional logic [12] and applications to the verification of quantum protocols [2, 3, 4, 11].

[1] M. Abadi and J. Y. Halpern, *Decidability and expressiveness for first-order logics of probability*, ***Information and Computation***, vol. 112 (1994), no. 1, pp. 1–36.

[2] J. C. Agudelo and W. A. Carnielli, *Paraconsistent machines and their relation to quantum computing*, ***Journal of Logic and Computation***, vol. 20 (2010), no. 2, pp. 573–595.

[3] E. Ardeshir-Larijani, S. J. Gay, and R. Nagarajan, *Equivalence checking of quantum protocols*, ***Proceedings of the 19th International Conference on Tools and Algorithms for the Construction and Analysis of Systems*** (Rome, Italy), (N. Piterman and S. A. Smolka, editors), vol. 7795 LNCS, Springer, 2013, pp. 478–492.

[4] P. Baltazar, R. Chadha, and P. Mateus, *Quantum computation tree logic: Model checking and complete calculus*, ***International Journal of Quantum Information***, vol. 6 (2008), no. 2, pp. 219–236.

[5] S. Basu, R. Pollack, and M.-F. Roy, ***Algorithms in Real Algebraic Geometry***, Springer, 2003.

[6] G. Birkhoff and J. von Neumann, *The logic of quantum mechanics*, ***Annals of Mathematics***, vol. 37 (1936), no. 4, pp. 823–843.

[7] R. Chadha, P. Mateus, A. Sernadas, and C. Sernadas, *Extending classical logic for reasoning about quantum systems*, ***Handbook of Quantum Logic and Quantum Structures: Quantum Logic*** (D. Gabbay K. Engesser and D. Lehmann, editors), Elsevier, 2009, pp. 325–372.

[8] M. L. D. Chiara, R. Giuntini, and R. Greechie, ***Reasoning in Quantum Theory***, Kluwer, 2004.

[9] C. Cohen-Tannoudji, B. Diu, and F. Laloë, ***Quantum Mechanics***, John Wiley, 1977.

[10] D. J. Foulis, *A half-century of quantum logic. What have we learned?*, ***Quantum Structures and the Nature of Reality*** (D. Aerts and J. Pykacz, editors), Kluwer, 1999, pp. 1–36.

[11] S. J. Gay, R. Nagarajan, and N. Papanikolaou, *QMC: A model checker for quantum systems*, ***Proceedings of the 20th International Conference on Computer Aided Verification*** (Princeton, USA), (A. Gupta and S. Malik, editors), vol. 5123 LNCS, Springer, 2008, pp. 543–547.

[12] P. Mateus, J. Ramos, A. Sernadas, and C. Sernadas, *Temporal logics for reasoning about quantum systems*, ***Semantic Techniques in Quantum Computation*** (I. Mackie and S. Gay, editors), Cambridge University Press, 2012, pp. 389–413.

[13] P. Mateus and A. Sernadas, *Exogenous quantum logic*, ***Proceedings of Workshop on Combination of Logics: Theory and Applications*** (Lisbon, Portugal), (W. A. Carnielli, F. M. Dionísio, and P. Mateus, editors), DMIST, 2004, pp. 141–149.

[14] P. Mateus and A. Sernadas, *Reasoning about quantum systems*, ***Proceedings on Logics in Artificial Intelligence*** (Lisbon, Portugal), (J. Alferes and J. Leite, editors), vol. 3229 LNCS, Springer, 2004, pp. 239–251.

[15] P. Mateus and A. Sernadas, *Weakly complete axiomatization of exogenous quantum propositional logic*, ***Information and Computation***, vol. 204 (2006), no. 5, pp. 771–794.

[16] M. A. Nielsen and I. L. Chuang, ***Quantum Computation and Quantum Information***, Cambridge University Press, 2000.

[17] A. Tarski, ***A Decision Method for Elementary Algebra and Geometry***, University of California Press, 1951.