Fibring Logics*

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Logics of a combined nature were abundant in the literature when, in the mid 1990s, the study of general mechanisms for combining logics developed into a well-posed research area [5, 6]. Several motivations, both theoretical and applied, concur to justify such a new line of research, but the increasing number of ever more complex logics and logical features appearing in application areas ranging from software engineering to linguistics was certainly amongst the most important. The idea of combining logics had been cooking in a low flame for more than a decade, namely in the particular context of modal logic [32, 63, 64, 74, 50, 29], and within the theory of institutions, with emphasis on equational logic [43, 56]. However, Dov Gabbay's 1996 article *Fibred Semantics and the Weaving of Logics - Part I: Modal and Intuitionistic Logics* [34], included in this anthology, brought the whole enterprise to a new era. There are two principal reasons to justify the impact of the paper. First, among various application examples, the paper put forward the notions underlying the general mechanism for combining logics known as *fibring*. Reinterpreting the original phrasing, the general problem of fibring two logics $\mathcal{L}_1$ and $\mathcal{L}_2$ is:

(P0) Characterize the logics $\mathcal{L}$ built over the combined language that conservatively extend the two, and in particular the minimal such logic $\mathcal{L}_1 * \mathcal{L}_2$.

Perhaps even more importantly, the paper also clearly outlined for the first time the main objectives and subproblems that should guide a systematic study of the general problem, namely:

(P1) Characterize the notion of a logical system.

(P2) Present methodologies for combining any two logics.

(P3) Investigate transfer properties.

(P4) Compare the combined logics obtained with known logics.

(P5) Study possible interactions between the logics being combined.

*This work was partially supported by FCT and EU FEDER, namely via the KLog project PTDC/MAT/68723/2006 of SQIG-IT. The authors are indebted to many colleagues with whom they have collaborated, over the years, on the topic of combining logics.

The specialized study of fibred modal logics, from fusions to products, has followed its own course [30, 78, 38, 52, 39, 79, 40, 46, 55, 41, 42, 48, 77], and we do not go through it here. Instead, we concentrate on the development of the general theory of combining logics, as entailed by the idea of *universal logic* [4], and which subsumes the work on combining institutions [57, 58, 11, 10, 12, 26]. At the light of problem (**P0**), we briefly describe the work in each of the five directions mentioned. According to our view of the field, we first analyze subproblems (**P1**) and (**P2**), then we look into subproblems (**P3**) and (**P5**), and last we tackle subproblem (**P4**). We conclude with a short appreciation of the paths pursued and those that lay ahead.

Methodology

The statement of (**P0**) is relatively ambiguous, its precise meaning lying in the possible answers to subproblems (**P1**) and (**P2**). For fibring to be rigorously defined, one must first agree on what is a logical system, including the structure of its syntax, in order to make precise what it means to write combined formulas by mixing constructors from different logics. Moreover, one needs to settle what it means for a logic to extend another, namely in a conservative way. The task suggests taking an algebraic, or even categorial, approach to the fibring operation [68, 17, 9]: identifying the class of mathematical objects one wants to combine, and then defining and studying the operation that combines two such objects into a third one.

What is a logical system? There are many conceptions of what a logic system may be [33], a question prone to many philosophical wanderings. Even adopting a modern view of logic based on consequence operators, after Tarski or Scott, there is still room for large variations depending on the richness of the underlying language (eg. sortedness and structurality), or on the underlying way of presenting them, namely semantically (eg. using appropriate notions of model and formula satisfaction), or deductively (eg. via Hilbert style axiomatizations, sequents or tableaux). Fortunately, a definitive answer is not necessary here. As long as one rigorously characterizes a collection of objects that one is willing to classify as logical systems, combining them makes perfect sense. Different notions of logical system, some very abstract, others quite particular, have been addressed and combined.

Combination mechanisms From the begining there was an informal but robust notion of combining deductive calculi, namely axiomatizations, but only a mechanical definition of fibred semantics stemming from the work on modal-based logics, as introduced in [34]. Fibred Kripke-like semantics was then rigorously defined in [68, 80], and later extended to general logical matrices and abstract structural consequence operators in [36, 7], still over unsorted (propositional based) languages. The scope of the fibring operation was then extended to logical systems with more general many-sorted languages [69, 80, 61], as well as to other useful formalisms, including labelled calculi and tableaux [25, 3, 62], logics represented as theories in a meta-logic [8, 22, 19] and higher-order logic [23]. An important variation of the general fibring construction occurs when one takes in consideration the possibility that the logics being combined may share some of the syntactic constructors, leading to notions like *constrained fibring* [68] or *dovetailing* [34]. As hinted above, it is relatively easy to deal with these issues in deductive settings. However, fibring semantic-based logical systems is much harder, and these questions raise very interesting and deep problems in connection with the meta-properties of the fibring operation, that we will discuss below.

Besides fibring, in its full generality, some attention has also been given to interesting and useful restrictions of the main notion. Simpler forms of combining logics, with less ambitious syntactic aims and restricted forms of mixing syntactic constructors, were studied in detail, namely *temporalization* [29], *synchronization* [66, 67], *parameterization* [16], *exogenous enrichment* [54], and *approximation* [31].

Metatheorems

Once the methodological questions are dealt with and fibring has been rigorously defined over a given class of logical systems, studying the properties of the fibring operation seems to be the obvious next step. Which properties transfer from $\mathcal{L}_1$ and $\mathcal{L}_2$ to $\mathcal{L}_1 * \mathcal{L}_2$? Or better, under which general sufficient conditions on the logics $\mathcal{L}_1$ and $\mathcal{L}_2$ can one guarantee that $\mathcal{L}_1 * \mathcal{L}_2$ will enjoy a certain property? Indeed, subproblem (**P3**) is deeply related to subproblem (**P5**), as preservation results depend strongly on the amount of interaction allowed (or required) between the logical systems being combined. In fact, ultimately, these investigations lead to another fundamental question. How does the combined logic $\mathcal{L}_1 * \mathcal{L}_2$ relate, in general, to the logics one started with?

Preservation results Already mingled in ancestor papers like [50], as well as in [34], one can find several transference results and non-preservation counterexamples about several interesting meta-logical properties, although just in the context of fibred modal logics. With the development of the general theory of combined logics, a great effort has been put in proving general transference results of meaningful properties, including, axiomatizability [68], algebraizability [47, 27, 28], and interpolation [21]. As explained above, the problems of preserving properties by fibring are particularly interesting but also substantially harder when one considers logical systems with rich multi-sorted languages with shared constructors. Special attention has been dedicated to the fundamental difficulties posed by semantical considerations, and their connection to the deductive ones, and therefore to finding general sufficient conditions for the preservation of soundness and completeness [80, 7, 72].

Conservativeness A fundamental case of preservation problem concerns logical derivations. To what extent are the inferences allowed in each of the ingredient logics $\mathcal{L}_1$ and $\mathcal{L}_2$ preserved to the fibred logic? Is $\mathcal{L}_1 * \mathcal{L}_2$ a conservative extension of the original systems? This desideratum is explicitly mentioned in the statement of the main problem (**P0**). However, as long as one allows the ingredient logics to share syntactic constructors, it becomes clear that it is not reasonable nor desirable to demand conservativeness to hold, in general. The fibring construction must, however, provide a combined logic which is "as conservative as possible". Such concerns are of prime importance in dealing with a particular instance of failure of conservativeness that became known as the *collapsing problem* [34, 35]. As mentioned above, this study motivated important developments and generalizations of the original definition of fibring for semantic-based logical systems, most notably using logical matrices, including *modulated fibring* [73] and *cryptofibring* [14].

Applications

The whole abstract idea of combining logics in an algebraic way is very appealing but, even if it is very well motivated, the question posed by subproblem (**P4**) is of utmost importance. Of course, having a hammer in hand and knowing how it works one should of course try it on a few nails. Still, this subproblem should be interpreted in a much deeper way. Are the general methods and results applicable and useful when one needs to build and study a logical system suitably combining features coming from several simpler logics? How do these features, even if thoroughly understood in isolation, interact with each other when combined? These are actually very difficult and challenging questions. Of course modal logics are an excellent playground for experimentation. However, there, specific results have been obtained that go much farther than the ones obtained in general for fibring. Anyway, things seem to be quite ok if one considers deductive formalisms, or logics represented in more general logical frameworks, as the typical tasks of putting together axioms and inference rules, even when there are interactions, are omnipresent in modern logic. However, in semantic-based formalisms, there is still a long way to go. There are promising steps forward, like [44], but collapsing phenomena have highlighted defects that can only be circumvented by further developing the theory of modulated fibring and cryptofibring, where one can aim at obtaining a better grip on the semantics of combined logics, while also opening room for less demanding general preservation results [73, 12, 14, 13].

Promising applications have been successfully addressed using simpler restricted combination mechanisms, namely temporalizations[49, 2] and exogenous enrichments [53]. The field of software engineering (where it is frequently necessary to work simultaneously with specifications written in different logics) was and remains the main target of application. However, most of the theoretical results on fibring fall short of this goal, namely because most were obtained for propositional-based logics, assume the homogeneous scenario for fibring (combining calculi of the same kind) and deal mainly with well behaved, structural logics. Artificial intelligence is another promising field of application [51, 45].

Outlook and final remarks

With the applications in mind, further work is needed towards expanding the theory of fibring towards more complex, possibly non-structural, logics, preferably within heterogeneous scenarios. The latter are considered in [24]. A promising approach to encompassing more general logics relies on graph-theoretic presentations and uses an abstraction map for relating models with the language [70, 71], thus avoiding the traditional homomorphic semantics underlying most of the previous work on fibring.

On another front, only a few transference results have been obtained so far. Much work is needed concerning the preservation by fibring of other interesting properties of logics, namely decidability, for instance via the preservation of the finite model property.

Given the recent progress in the related theory and applications of combining first-order theories [59, 60, 75, 76, 1], it seems worthwhile to try to establish a bridge between the two fields and thus open the door for a possible interplay between abstract model theory and the theory of fibring.

In order to contain the size of the list of bibliographic references, we refer instead the reader to recent surveys [65, 15, 9, 18] and books [37, 20]. As ever, Gabbay's book [36] is also mandatory reading to this end.

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