Dov M. Gabbay, *Fibring Logics*, Oxford Logic Guides 38, Oxford University Press, New York, 1999, pp. xiii + 475, ISBN 0-19-850381-4

Interest in the problem of combining logics has been growing in the last decade, driven by foundational issues and applications [BdR97]. The theoretical interest in the problem is clear. Every logician will have looked at some logic as a combination of two simpler logics. For instance, one is tempted to look at predicate modal logic as resulting from the combination of first order logic and propositional modal logic. But the approach will be significant if and only if general transference (or preservation) results are available about the mechanism used for combining the logics. For example, if it has been established that completeness is preserved by mechanism $\bullet$ and it is known that logic $\mathcal{L}$ is given by $\mathcal{L}' \bullet \mathcal{L}''$, then the completeness of the combination $\mathcal{L}$ follows from the completeness of $\mathcal{L}'$ and $\mathcal{L}''$. No wonder that much effort has been dedicated to establishing such transference results and/or counterexamples for mechanisms for combining logics. The practical interest in the problem is also clear, at least from the point of view of those working in recent application areas of mathematical logic, like knowledge representation (within artificial intelligence) and formal specification and verification (within software engineering). Indeed, in these application areas, the need for working with several formalisms (read logics) at the same time is the rule rather than the exception. For instance, in a knowledge representation problem it may be necessary to work with both temporal and deontic knowledge. And in a software specification problem it may be necessary to work with both equational and temporal specifications. In such applications, theoretical results on the problem of combining logics are of immediate relevance.

Among the combination mechanisms, fibring has deserved special attention because it is quite general. But what is fibring? For those not familiar with the concept, a (very) rough description is possible in a few sentences.

At the syntactic level, the language of the fibring is obtained by the free use of the language constructors (atomic symbols and connectives) from both given logics. So, for example, when fibring a temporal logic and a deontic logic, "mixed" well formed formulae like $((G\alpha) \Rightarrow (O(F\beta)))$, stating "if always α then obligatorily some time β", appear in the resulting logic. Naturally, in many cases one wants to share some of the symbols. The previous example involves the "constrained" form of fibring imposed by sharing the propositional symbols (variables and connectives).

At the deductive level, provided that the two given logics are endowed with deductive systems of the same type, the deductive system of the fibring will be obtained by the free use of the inference rules from both. This approach will be of interest (having in mind subsequent proof of preservation of completeness) only if the two given deductive systems are "schematic" in the sense that their inference rules are "open" for application to formulae with "foreign" symbols. Fortunately, this schematic presentation of deductive systems is the rule rather than the exception in mathematical logic. For instance, when one defines *modus ponens* by the rule MP, $\{(\xi_1 \Rightarrow \xi_2), \xi_1\} \vdash \xi_2$, in some Hilbert system, one may implicitly assume that the instantiation of the "metavariables" ξ_1, ξ_2 by any formulae, possibly with symbols from both logics, is allowed when applying MP in the fibring.

The semantics of fibring is quite involved and, therefore, it is better to consider a special case. Consider only logics with a "propositional base", that is, with propositional variables and connectives of arbitrary arity. A possible, quite general, model for such a logic is provided by a triple $\langle U, \mathcal{B}, \nu \rangle$ where U is a set (of points, worlds, states, whatever), $\mathcal{B} \subseteq \wp U$, and $\nu(c) : \mathcal{B}^n \to \mathcal{B}$ for each c of arity $n \geq 0$. In what follows we consider only logics with a propositional base and a semantics given by a class of such triples. Given two such logics $\mathcal{L}', \mathcal{L}''$ what should be the semantics of their fibring? It will be a class of models of the form above, such that at each point $u \in U$ it is possible to extract a model from $\mathcal{L}'$ and one from $\mathcal{L}''$. If symbols are shared, the two extracted models should agree on them. In order to visualize the semantics of fibring, consider the fibring of a propositional linear temporal logic with a propositional linear space logic. Each model of the fibring will be a cloud of points where at each point one knows the time line and the space line crossing there. For instance, at the point $\langle$*Lisbon, 10h15m 10 September 1999*$\rangle$ one knows the time line (of past, present and future) of Lisbon and the space line (of the planet taken as line for the sake of the example) at that time.

From this description (albeit illustrated only for logics with a propositional base), it should be clear that fibring is quite general as a mechanism for combining logics. In the last decade, many results have been proved about it, it has started to have an impact on the application areas, and it was shown that it encompasses as special cases other interesting mechanisms for combining logics.

Dov Gabbay has been at the center of this movement from the beginning, and his research monograph *Fibring Logics* is a most welcome addition to the field. Furthermore, it is also of interest to those looking at fibring from outside. Any logician entering the field of combination of logics should start with this book. It is also mandatory reading to any one interested in applications of logic to artificial intelligence and software engineering, but a word of caution should be given. It is not a textbook, and the author presumes a lot from the reader in terms of previous familiarity with many aspects of mathematical logic.

Based to a large extent on expanded versions of papers by the author, the book constitutes a wonderful reference on fibring, including a detailed discussion of alternative formulations of the concept, many interesting cases (like modal, temporal and intuitionistic logics, as well as non-monotonic and fuzzy logics), several transference results for specific cases (like preservation of soundness, completeness, finite model property and decidability), different types of deductive systems (like Hilbert, tableau and labelled deductive systems, as well as intuitionistic logic programs), and many open problems. It also includes a detailed treatment of several (somewhat) related mechanisms for combining logics like self-fibring (of predicate logics), product (of modal logics) and temporalisation. The list of symbols, the table of contents and the word index are well done. The set of references is extensive but (already) unavoidably out of date in some details (a problem easily overcome by any one interested – use a citation index service starting with any of the clearly seminal papers on the subject, for instance [Gab96]).

The book provides the perfect state of the art in the topic and shows the need for further research, by pointing out many new interesting doors still to be closed (like a unified concept of fibring, more general preservation results, fibring of predicate and higher order logics, algebraic semantics of fibring, and encoding of fibring of deductive systems in logic frameworks).

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