Reputation-based Ranking Systems and their Resistance to Bribery

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Abstract

We study the bribery resistance properties in two classes of reputation-based ranking systems, where the rankings are computed by weighting the rates given by users with their reputations. In the first class, the rankings are the result of the aggregation of all the ratings, and all users are provided with the same ranking for each item. In the second class, there is a first step that clusters users by their rating pattern similarities, and then the rankings are computed cluster-wise. Hence, for each item, there is a different ranking for distinct clusters. We study the setting where the seller of each item can bribe users to rate the item, if they did not rate it before, or to increase their previous rating on the item. We model bribing strategies under these ranking scenarios and explore under which conditions it is profitable to bribe a user. Further, we provide the optimal bribing strategies in several cases. By clustering users and computing dedicated rankings to each cluster, we show that bribing, in general, is not as profitable as in the simpler case where no clustering is performed. Finally, we illustrate our results with experiments using both synthetic and real data.

I. Introduction

The evolution of our contemporary society towards an information economy boosted the development of e-commerce. Further, the fast pace of information spreading around the world has been reconfiguring our social interactions. The social networks and online *fora*, in general, promoted the exchange of opinions and potentiated the online word of mouth (WOM), which in turn gained potential to drive e-commerce sales, see [1], [2] and [3], for instances.

Nowadays, the importance of reviews and rankings became paramount for sellers, because the visibility and the sales numbers are related with them, [4], [5] and [6]. Furthermore, studies pointed out that, in several cases, online reviews may be more influential than traditional marketing, see [7]. This strong influence of online reviews on sales increased the attempts to manipulate them, [8]. Fostering the need for designing ranking systems robust to spam and attacks, as in [9], [10] and references therein.

The companies spend money in convincing users to vouch for their products/services, either by giving samples of their products so that users can comment on them, or by directly paying them to give positive feedback on their products and negative on competitor products, see [11].

Two central issues in social choice are the ones of lobbying and bribing in decision-making. Much work has been devoted to these problems, starting from [12]. In their work, the authors examined the effects between lobbying and legislative bargaining on policy formation. The main results they present are: the generated policies are not intermediate between policies that would come from both pure lobbying and pure bargaining; the policies are strongly skewed in favor of the agenda-setter; although the effect that lobbying has on policies, the equilibrium contributions are very small.

Aware of the importance and influence of both bribing and manipulation of rankings, we model these phenomena and characterize them in a quantitative way, so that their impact can be better understood. Further, we characterize ways to mitigate the impact of such behaviors, by showing that reputation-based ranking systems using clusters are, in general, more robust to bribery.

Previous work. The influence of individual decisions on global properties in network-based rating systems was studied in several works. In [13], the authors investigate how to turn a product into a tendency among users by changes on a social network. In [14], the authors explore how to design an impartial mechanism of peer review in order to mitigate the effect of a reviewer interfering with the likelihood of its own work being accepted.

Also, the computational complexity of those issues has gain considerable attention, and it is studied in works as [15], [16], [17], [18]. First, in [15], the authors analyzed the computational complexity of problems in social choice using parameterized complexity. They showed that lobbying, in conditions of "direct democracy", requires solving a computational hard problem. After, in [16], the authors showed that the complexity of bribery is extremely sensitive to the setting under analysis. They studied the bribery problem for several setups of election systems, and they designed a simple to check condition that separates those problems into NP-complete problems or problems in P. Whereas in [17], the authors studied the problem of judgment aggregation procedures and bribery in these paradigm. They showed that these problems are NP-hard, and that they are also hard to solve when assessing their parameterized complexity. Finally, in [18], the authors studied the problem of whether a lobby can choose voters to influence so that each of the lobby issues gets a majority of approval. They modeled this problem as a simple matrix modification problem and proved that lobbying is fixed-parameter tractable, providing a greedy approximation algorithm with a logarithmic-factor optimality gap for the lobbying problem.

In [10], the authors proposed a reputation-based ranking system that clusters users by their ranking pattern similarities. The authors showed that, by doing so, their approach is more robust to both spamming users and users which try to attack the ranking system in order to increase/decrease the ranking of a set of items.

The authors of [19], analyzed the resistance of two ranking systems, one that simply averages the ratings of users AA, another that takes into account the network of influence of a given user, using the AA to compute the ranking for each network. They showed that the AA ranking system is bribable, and, in particular, bribing users which did not rate is profitable. When considering social networks of users, they show that the bribery effect is diminished and under some conditions on the networks can even drop to zero. Their work assumes a static (fixed) set of users with only one item to rate. In both ranking systems that they study, the ranking is computed as the AA of the users' ratings. This choice implies giving the same relevance to all users, treating equally regular users, spamming users or users that want to tamper with the system. Also, the AA does not capture a possible multimodal behavior of ratings, as noticed in [20]. This motivated us to explore bribing in reputation-based ranking systems and, in particular, to explore the case where users belong to clusters which, intuitively, must diminish the bribing effect.

Our contribution. Here, we study the resistance to bribery of a class of ranking systems that assigns reputations to users. We show that a ranking system that computes the rank of an item by the weighted average of the users' ratings with their reputations is bribable. In particular, users with a reputation above the average of the reputations of users that rated the item are profitable to bribe. However, by clustering users by their rating pattern and assigning possibly different rankings to the same item for different clusters, we increase the bribery resistance of the ranking system. In other words, clustering users and presenting a dedicated ranking of items for each cluster as in [10] makes the ranking system much more robust to bribing. Intuitively, the more reputation a user has the larger is the profit of bribing this user. Further, for a user to be bribable its reputation needs to be larger than the average reputation of the users that rate the item. This bound increases in the clustering scenario, since within each cluster the number of users that rated the item is less (or equal in the worst case) than the non clustering scenario. Our model can also apply to evaluate marketing strategies, where a company is willing to invest money to boost its sales, either increasing the users base or boosting positive reviews.

Paper structure. In Section II, we set up the notation and needed definitions to develop our model and to analyze bribery. In Section III, we present the main results of the paper, we characterize under which conditions the bribing strategies are profitable, and we derive the optimal strategies. Further, we show that reputation-based ranking systems with clusters are more robust to bribery than simple ones (without clustering). We present examples with synthetic and real data, in Section IV. Section V concludes the paper drawing avenues for further research.

II. Preliminaries and definitions

To study the resistance to bribery properties of ranking systems we set up the some notational conventions and definitions. Reputation-based ranking systems assign weights to users in order to aggregate their rankings on given items. Here, we discuss two classes of ranking systems, namely, the *bipartite reputation-based ranking systems* and the *multipartite reputation-based ranking systems*, see [10].

Given a set of items I, suppose that any user u, among the set of users U, can assign a *rating*, R_{ui}, to any item $i \in I$. Further, for each item i suppose that we want to aggregate the scores of users using a weighted average. In this setting, the reputation-based ranking systems, attribute a *reputation* score, c_u, to every user $u \in U$, to weight the ratings of users. The *ranking*, r_i, for each item i is then computed as a function of the reputations of the users who rated the item and their reputations.

In this work, we assume, without loss of generality (w.l.o.g.), that both reputations and rankings take values in $]0, 1]$. If the former does not hold, we can always rescale the values to fulfill this condition.

A *bipartite reputation-based ranking system* iteratively computes the ranking of each item, using weighted average, whose coefficients are the users reputations. The reputations are, in turn, updated considering the discrepancy of the item's ranking and the user's rating on that item. This process is repeated until convergence. In this setting, every user, $u \in U$, has access to the same ranking, r_i, for each item, $i \in I$. Schematically, we may represent these raking systems as a bipartite graph, in which one set of vertices correspond to users and the other to items. We have edges connecting the vertices, weighted by the ratings that users gave to items, only allowing for user-item connections, see Figure 1. In this setting, to color vertices with common edges we need exactly two colors, hence the use of *bipartite* in the terminology, see [21]. For a definition, see [9] or [10].

A *multipartite ranking system* takes two steps. In the first step, the users are clustered by their rating pattern similarities. In the second step, the ranking of each item is iteratively computed, as in the *bipartite reputation-based ranking system* case, using only information from each cluster, producing (possibly) different rankings for different clusters. Since the ranking of an item may differ from cluster to cluster, every user on the same cluster access the local ranking of a given item, in the case where the item was rated by, at least one user belonging to that cluster. If, for a given item, within a given cluster, no user

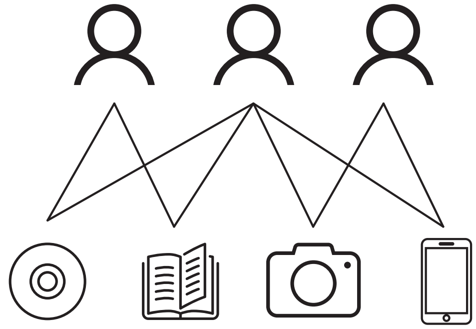

Fig. 1: Bipartite graph representation of users and items with edges interconnecting them with weights given by the ratings that users gave to items.

rated that item, the available ranking for that item is the weighted average of that item's rankings among clusters where the item was rated. We can represent this ranking system as a multipartite graph. Each user can then be connected to several items, with edges weighted by their ratings as before. We now allow for edges between users (encoding similarities between them), where connected users form a cluster. This construction forms a multipartite graph, since we need more than two colors in order to color vertices with common edges with different colors, see Figure 2 and for the definition see [21]. For further details

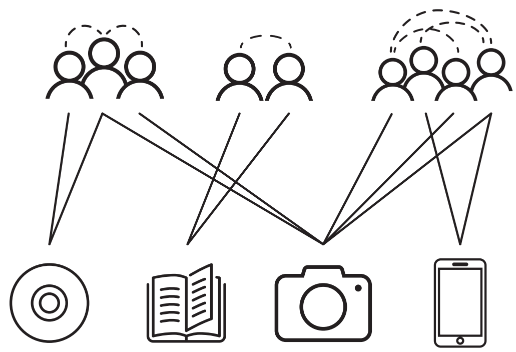

Fig. 2: Multipartite graph representation of users and items with edges interconnecting them with weights given by the ratings that users gave to items. Each cluster has users connected with dashed edges, representing which users are similar.

on properties and convergence of this class or ranking systems see [10].

In the bipartite case, the *ranking of an item* is computed as:

$$r_i = \frac{1}{\alpha} \sum_{u \in U_i} c_u R_{ui}, \quad \alpha = \sum_{u \in U_i} c_u, \tag{1}$$

where α is the sum of reputations of the users that rated item i, and U_i denotes the set of users that rated item i.

In the *multipartite ranking systems* case, let $\mathcal{M}$ denote the set of users, and $\mathcal{M}_1, \dots, \mathcal{M}_N$ be a partition of $\mathcal{M}$ into N disjoint groups of users, that is:

$$\mathcal{M} = \bigcup_{n=1}^{N} \mathcal{M}_n \text{ and, for } m \neq n,\ \mathcal{M}_m \bigcap \mathcal{M}_n = \varnothing. \tag{2}$$

We denote the set of items rated by users that belong to the cluster $\mathcal{M}_n$ by $I^{\mathcal{M}_n}$, with $I^{\mathcal{M}_n} = \bigcup_{u \in \mathcal{M}_n} I_u$, where I_u denotes the set of items rated by user u. The set of users in the cluster $\mathcal{M}_n$ that rated item i is denoted by $U_i^{\mathcal{M}_n}$, where $U_i^{\mathcal{M}_n} = U_i \cap \mathcal{M}_n$. In this case, the ranking is computed independently for each cluster as

$$r_i^{\mathcal{M}_n} = \frac{1}{\alpha} \sum_{u \in U_i^{\mathcal{M}_n}} c_u R_{ui}, \quad \alpha = \sum_{u \in U_i^{\mathcal{M}_n}} c_u. \tag{3}$$

Recall that for users belonging to a cluster $\mathcal{M}_n$, the displayed ranking of item i can be one of the following two possibilities: (i) the ranking of the item for that cluster $r_i^{\mathcal{M}_n}$, whenever there are users in the cluster that rated item i; (ii) otherwise, the ranking of item i is the weighted average of the rankings of i for the clusters with users that rated item i, that is:

$$\bar{r}_i = \sum_{n \in \mathcal{X}_i} \left| U_i^{\mathcal{M}_n} \right| r_i^{\mathcal{M}_n} \Big/ \sum_{n \in \mathcal{X}_i} \left| U_i^{\mathcal{M}_n} \right|,$$

where $\mathcal{X}_i = \{m : i \in I^{\mathcal{M}_m} \text{ and } m = 1, \ldots, N\}$. In what follows, for a set of users $U' \subseteq U$, we denote by $\bar{c}_{U'} = \sum_{u \in U'} c_u / |U'|$.

Suppose the seller of item i has an initial wealth that is proportional to the popularity of the item, and the number of costumers that rated the item. To further boost the sales of item i, the seller may invest his resources (wealth) to promote the popularity of the item so that users like it more, and/or to expand his consumer base by making people buy it, and like it, but not necessarily love it. Here, we model this scenario assuming that the popularity is an increasing function of the ranking of the item, r_i, and supposing that the number of consumers that bought the item is an increasing function of the number of users that rated the product, $|U_i|$.

We define the *reward function* or *wealth*, in the bipartite and multipartite cases, for seller of item i as

$$J_i = |U_i| \, r_i \text{ and } \bar{J}_i = \sum_{n \in \mathcal{X}_i} J_i^{\mathcal{M}_n}, \tag{4}$$

respectively, where $J_i^{\mathcal{M}_n} = \left| U_i^{\mathcal{M}_n} \right| r_i^{\mathcal{M}_n}$.

We define the *strategy* of the seller of item i as a vector $\sigma^i \in \mathcal{S}_i$, with size $|U|$, where the u-th entry is the value of the invested wealth to convince user u to increase his rating by ρ_u, and $S_i \subseteq [0,1]^{|U|} \backslash \{\mathbf{0}\}$, where $\mathbf{0}$ is the null strategy that does not bribe any user. Whenever user u rated item i with R_{ui} the value of the strategy must satisfy $\rho_u \leqslant 1 - R_{ui}$. If $\rho_u = 0$ this means the seller does not try to persuade user u to rate or to change his rating item i.

For the seller of item i, we denote by $\Xi_i = \{\sigma^i \in \mathcal{S}_i : \sigma^i(u) = \rho_u = 0 \text{ for all } u \notin U_i\}$ the set of strategies that consists, exclusively, in bribing users that already rated the item i. Analogously, we denote by $\bar{\Xi}_i = \mathcal{S}_i \backslash \Xi_i = \{\sigma^i \in \mathcal{S}_i : \sigma^i(u) = \rho_u = 0 \text{ for all } u \in U_i\}$, the set of strategies that consists in bribing users that did not rate item i. We say that a bribing strategy σ^i is an *elementary strategy* whenever for some user $u \in U$ we have that $\sigma_u^i > 0$ and for all $v \in U$ with $v \neq u$ we have that $\sigma_v^i = 0$. To simplify notation, instead of using $\sigma^i(u)$ to denote the strategy of item i seller to bribe user u, we write σ_u^i. Further, the wealth spent by strategy σ^i is given by $\|\sigma^i\|_1 = \sum_{u \in U} \sigma_u^i$.

After playing strategy σ^i, the wealth of seller i, in the bipartite and the multipartite cases, becomes:

$$J_{\sigma^i} = |U_{\sigma^i}| \, r_{\sigma^i} - \sum_{u \in U_{\sigma_i}} \rho_u, \; \bar{J}_{\sigma^i} = \sum_{n \in \mathcal{X}_i} \left| U_{\sigma^i}^{\mathcal{M}_n} \right| r_{\sigma^i}^{\mathcal{M}_n} - \sum_{u \in U_{\sigma_i}} \rho_u,$$

respectively, where r_{σ_i} denotes the new value for r_i after playing strategy σ^i.

The *profit* of playing the strategy σ^i is then

$$\pi_{\sigma^i} = J_{\sigma^i} - J_i \text{ and } \bar{\pi}_{\sigma^i} = \bar{J}_{\sigma^i} - \bar{J}_i, \tag{5}$$

respectively for the bipartite and multipartite scenarios.

III. Bribing in ranking systems

Here we study the resistance to bribery of reputation-based ranking systems that compute items' ranking by averaging the users ratings with their reputations. As we mentioned above, some of the ranking systems use iterative procedures to update the rankings and users' reputations. In order to simplify the analysis we assume an initial assignment of reputations to users, which corresponds to use an iterative reputation based ranking systems only once, to determine the initial reputation of each user.

Next, we consider the bipartite and multipartite ranking systems, and we study its behavior against bribing strategies. First, we characterize the set of decomposable bribing strategies and also provide conditions under which the strategies are not decomposable. After, we find the conditions for the different strategies to be profitable. Moreover, we characterize the optimal bribing strategies, by encoding the problem of computing them as a linear programming (linear optimization) problem that can be solved by algorithms with polynomial time complexity [22]. Further, in some cases, we can actually describe the solution of the optimal strategy in closed form. Finally, we compare both bipartite and multipartite cases and show that using clusters makes the ranking system more robust to bribery, in general.

A. Properties of strategies and its profit in the bipartite case

First, we investigate whether a strategy is decomposable into elementary ones, or, if not, what particular conditions allow us to decompose a strategy. This comes in hand when studying complex strategies that consist in bribing several users at once. We start by considering the case where the sellers of item $i \in I$ bribe users that already rated the item. In this case, we prove that all strategies bribing several users at once are decomposable into several elementary strategies.

Proposition 1. *Let $u, v \in U_i$ be two users that rated the item $i \in I$. Consider two strategies, σ_u^i and σ_v^i, that consists in bribing the users to change their ratings from R_{ui} and R_{vi} to $R_{ui} + \rho_u$ and $R_{vi} + \rho_v$, respectively. In these conditions, we have that $\pi_{\sigma_u^i + \sigma_v^i} = \pi_{\sigma_u^i} + \pi_{\sigma_v^i}$.*

Proof. When the seller of item i plays the strategy σ_u^i, the ranking of item i changes according to

$$r_{\sigma_u^i} = r_i + \frac{c_u}{\alpha}\rho_u, \tag{6}$$

where $\alpha = \sum_{v \in U_i} c_v$. Thus, an elementary strategy's profit is:

$$\begin{aligned} \pi_{\sigma_u^i} &= |U_i|\, r_{\sigma_u^i} - \rho_u - |U_i|\, r_i \\ &= \left(\frac{c_u}{\alpha}|U_i| - 1\right)\rho_u = \left(\frac{c_u}{\bar{c}_{U_i}} - 1\right)\rho_u. \end{aligned} \tag{7}$$

The profit of the sum of strategies, $\sigma_u^i + \sigma_v^i$, is given by:

$$\begin{aligned} \pi_{\sigma_u + \sigma_v} &= |U_i|\, r_{\sigma_u^i + \sigma_v^i} - (\rho_u + \rho_v) - |U_i| r_i \\ &= |U_i| \frac{(c_u\rho_u + c_v\rho_v)}{\alpha} - (\rho_u + \rho_v) \\ &= \pi_{\sigma_u} + \pi_{\sigma_v}, \end{aligned}$$

which is equal to the sum of the profits of the elementary strategies, σ_u^i and σ_v^i. □

Remark 1. *The case where we first bribe a user u to change its rating from R_{ui} to $R_{ui} + \rho_u$ and then to $R_{ui} + \rho_u + \rho'_u$ is the same when we bribe the user to change its rating from R_{ui} directly to $R_{ui} + (\rho_u + \rho'_u)$.*

We now consider the case when a seller opts to bribe users that did not rate the item i.

Proposition 2. *Consider a user that did not rate the item i,* i.e.*, $u \notin U_i$, and any other user $v \in U$. The strategy that consists in bribing both users, u and v, does not carry the same profit as the sum of the profits of bribing each user,* i.e.*, $\pi_{\sigma_u^i + \sigma_v^i} \neq \pi_{\sigma_u^i} + \pi_{\sigma_v^i}$, unless both elementary strategies have zero profit.*

Proof. If both users did not rate the item i, their strategies change the ranking of the product in the same way:

$$r_{\sigma_u^i} = \frac{\sum_{v \in U_i} c_v R_{vi} + c_u\rho_u}{\alpha + c_u} = \frac{\alpha r_i + c_u\rho_u}{\alpha + c_u}. \tag{8}$$

Thus, using the definition of profit in (5), it generates a profit of

$$\begin{aligned} \pi_{\sigma_u^i} &= (|U_i| + 1)\frac{\alpha r_i + c_u\rho_u}{\alpha + c_u} - \rho_u - |U_i|\, r_i \\ &= (\alpha - |U_i|\, c_u)\frac{r_i - \rho_u}{\alpha + c_u}. \end{aligned} \tag{9}$$

Hence, the profit for the sum of strategies, $\sigma_v^i + \sigma_u^i$, which bribes both users is

$$\begin{aligned} \pi_{\sigma_u^i + \sigma_v^i} &= \left|U_{\sigma_u^i + \sigma_v^i}\right| r_{\sigma_u^i + \sigma_v^i} - (\rho_u + \rho_v) - |U_i|\, r_i \\ &= (|U_i| + 2)\left[\frac{\alpha r_i + c_u\rho_u + c_v\rho_v}{\alpha + c_u + c_v}\right] - (\rho_u + \rho_v) - |U_i|\, r_i \\ &= \frac{\alpha + c_u}{\tilde{\alpha}}\pi_{\sigma_u^i} + \frac{\alpha + c_v}{\tilde{\alpha}}\pi_{\sigma_v^i} + \frac{1}{\tilde{\alpha}}(\rho_u - \rho_v)(c_u - c_v), \end{aligned}$$

where $\tilde{\alpha} = \alpha + c_u + c_v$. To have a positive profit of the sum of strategies that is equal to the sum of the profits of each elementary strategy, we need the following conditions to hold

$$\frac{\alpha + c_u}{\tilde{\alpha}} = \frac{\alpha + c_v}{\tilde{\alpha}} = 1 \text{ and } (\rho_u - \rho_v)(c_u - c_v) = 0,$$

this implies $c_u = c_v = \tilde{\alpha} - \alpha > 1$, which contradicts the fact that $c_u, c_v > 0$. However, in the case that $c_u = c_v = \bar{c}_{U_i}$ we have that the profits of the elementary strategies are zero, and the profit of the sum of the strategies is also zero. The case where one of the users to be bribed did not rate the item, $v \notin U_i$, but the other user did, $u \in U_i$, yields a profit of

$$\begin{aligned}
\pi_{\sigma_u^i+\sigma_v^i} &= \left|U_{\sigma_v^i}\right| r_{\sigma_u^i+\sigma_v^i} - (\rho_u + \rho_v) - |U_i|\, r_i \\
&= (|U_i| + 1)\, \frac{\alpha r_i + c_u\rho_u + c_v\rho_v}{\alpha + c_v} - (\rho_u + \rho_v) - |U_i|\, r_i \\
&= \frac{|U_i|}{\alpha + c_v} c_v(\rho_v - r_i) + \frac{|U_i|}{\alpha + c_v} c_u\rho_u \\
&+ \frac{\alpha}{\alpha + c_v}(r_i - \rho_v) + \frac{1}{\alpha + c_v}\rho_u(c_u - c_v) - \frac{\alpha}{\alpha + c_v}\rho_u \\
&= (\alpha - |U_i| c_v)\frac{r_i - \rho_v}{\alpha + c_v} + \frac{\alpha}{\alpha + c_v}\rho_u \left(\frac{c_u}{\alpha}|U_i| - 1\right) \\
&+ \frac{1}{\alpha + c_v}\rho_u(c_u - c_v) \\
&= \frac{\alpha}{\alpha + c_v}\pi_{\sigma_u^i} + \pi_{\sigma_v^i} + \frac{1}{\alpha + c_v}\rho_u(c_u - c_v).
\end{aligned} \tag{10}$$

Which carries the same conclusion as above. □

As we noted in the previous proof, there are special conditions on the reputation of users that makes the profit to be zero, hence decomposable into elementary strategies. We discuss this situations in the next result.

Proposition 3. *Pick any item $i \in I$. Consider the following cases: The seller of i choses to bribe users that already rated the item i, $u, v \in U_i$, and all the users have the same reputation $c_u = c_v = \bar{c}_{U_i}$. In the above case, the strategy is not profitable, and the sum of the profits of each elementary strategy has also zero profit, $\pi_{\sigma_u^i+\sigma_v^i} = \pi_{\sigma_u^i} + \pi_{\sigma_v^i} = 0$.*

Proof. For the composition of the strategies the profit is given by (10), with $c_v = c_w = \bar{c}_{U_i}$, thus $\pi_{\sigma_v^i+\sigma_w^i} = 0$. The strategy σ_v^i has profit given by (7), where $c_v = \bar{c}_{U_i}$, hence $\pi_{\sigma_v^i} = 0$. Also, the strategy σ_w^i has the profit given by (9), where $c_w = \bar{c}_{U_i}$, hence $\pi_{\sigma_w^i} = 0$. □

Note that this marks a difference from the work in [19], where, in Example 1, the authors show that with the ranking computed by the AA a user that did not rate the item can be bribed with an increase of wealth.

Next, we analyze strategies in terms of the profit they carry, because want to classify users into bribable and non-bribable ones, based on their reputation. We assume that all information is publicly available to sellers, both users' ratings and reputations. First, we study bribery on users that already rated the item whose sellers want to increase the ranking.

Proposition 4. *If user v rated item i, $v \in U_i$, an admissible strategy, $\sigma^i \in \Xi_i$, such that (s.t.) $\|\sigma^i\|_1 = \sigma_v^i = \rho_v$, is profitable whenever $c_v > \bar{c}_{U_i}$.*

Proof. Since, $\rho_v > 0$, the profit of such strategy, σ_v^i is given by (7), which is positive whenever $c_v > \frac{\alpha}{U_i} = \bar{c}_{U_i}$. □

We obtain, as a corollary of Proposition 4, the result of Lemma 2 in [19]. In fact, the proof is the computation of the reward of σ_v^i, in the proof of Proposition 3.

Corollary 1. *If $v \in U_i$, and $c_v = c_u = \bar{c}_{U_i}$ for all $u \in U_i$, i.e. the ranking is computed by the arithmetic average, then $\pi_{\sigma_v^i} = 0$.*

Suppose a seller of an item, say i, wants to bribe a user that did not yet rate that item. We explore what conditions make this action profitable.

Proposition 5. *Let $v \notin U_i$, the strategy σ_v^i is profitable whenever one of the following holds:*

1) $c_v < \bar{c}_{U_i}$ *and* $\rho_v < r_i$,
2) $c_v > \bar{c}_{U_i}$ *and* $\rho_v > r_i$.

Proof. The result follows from (9). □

Notice that, the result of Proposition 5 means that, in case 1), if a seller bribes a user (that did not rate item i) that has reputation below the average then the bribing value, ρ_v, must be smaller than r_i. This is because, the effect of bringing a new rated to the set of raters increase the wealth, as long we do not pay a high price, ρ_v, since the reputation of the user is smaller, henceforth the effect on the rating is small. In the case where the bribed user has a reputation above the average, case 2), its effect on the ranking of the item is large, therefore bribing with a value below the ranking would degrade it, thus decreasing the wealth.

We obtain as a corollary that, when the ranking is computed by the arithmetic average, it is not profitable to bribe a user that did not rate the item. We explore these results with examples, in Section IV.

B. Optimal Strategies in the bipartite scenario

Here, we investigate what is the optimal investment strategy that a seller of item i should take to increase his initial wealth, by influencing the opinion of costumers. We first consider the simpler cases, where a seller either tries to influence the opinion of costumers that already rated item i, or tries to persuade potential costumers that did not rate item i. Then, we proceed to analyze the more complex case when a seller has the option to influence both raters and non-raters. In this setting we obtain the optimal bribing strategies in closed form.

To model these problems, we consider a common set up, that we now detail. We assume that the seller of item i has an initial wealth of J_i, recall (4),and we consider two reference costumers, u and v, with reputations $c_u > c_v$. Our aim is to compute the profit per amount of invested wealth, $\frac{\pi_\sigma}{\|\sigma\|_1}$, so we can design the optimal bribing strategies.

a) Bribing users that already rated item i: Let us consider the case where the seller wants to bribe users that already rated item i, *i.e.* $u \in U_i$. We formulate this problem as

$$\begin{cases} \text{maximize:} & \pi_{\sigma^i} \\ \text{subject to:} & \|\sigma^i\|_1 \leqslant J_i \\ & \sigma^i \in \Xi_i \end{cases}$$

As we show in Proposition 4, to have a positive profit $\pi_{\sigma^i_u}$, when bribing user u, we need to have $c_u > \frac{\alpha}{|U_i|} = \bar{c}_{U_i}$. Therefore, we do not consider strategies that bribe users, v, s.t. $c_v < \bar{c}_{U_i}$, since it would imply not increasing the wealth, J_i.

Let $c_u > c_v > \bar{c}_{U_i}$, we look into the profit per unit of invested resources, $\pi_{\sigma^i_u}/\rho_u - \pi_{\sigma^i_v}/\rho_v = (c_u - c_v)/\bar{c}_{U_i} > 0$. Hence, the profit per unit of invested wealth is larger for user u than for user v. The optimal strategy is then: to bribe users by decreasing order of their reputation, investing all the available wealth until either the exhaustion of available profitable users ($c_u > \bar{c}_{U_i}$) or the exhaustion of funds to bribe profitable users.

b) Bribing users that did not rate the item i before: Under the same conditions for seller of item i, suppose that he wants to bribe users that did not rate the item, *i.e.*, $u \notin U_i$. We formulate this problem as

$$\begin{cases} \text{maximize:} & \pi_{\sigma^i} \\ \text{subject to:} & \|\sigma^i\|_1 \leqslant J_i \\ & \sigma^i \in \bar{\Xi}_i \end{cases}$$

Let users $u, v \notin U_i$ be s.t. $c_u > c_v$, and let $\alpha = \sum_{w \in U_i} c_w$, $\gamma = \frac{|U_i| c_u - \alpha}{c_u + \alpha}$ and $\delta = \frac{|U_i| c_v - \alpha}{c_v + \alpha}$. The profit is given by (9), hence, we have for user u and v

$$\frac{\rho_u - r_i}{c_u + \alpha} \left(|U_i| \, c_u - \alpha \right) \text{ and } \frac{\rho_v - r_i}{c_v + \alpha} \left(|U_i| \, c_v - \alpha \right),$$

respectively. The difference of profits is $(\rho_u - r_i)\gamma - (\rho_v - r_i)\delta$, and hence for the same amount of spent wealth, $\pi_{\sigma^i_u}/(\rho_u - r_i) > \pi_{\sigma^i_v}/(\rho_v - r_i)$, because $\gamma > \delta$.

Once again, the optimal strategy is to bribe users by decreasing order reputation, investing all the available wealth until either the exhaustion of profitable users ($c_u > \bar{c}_{U_i}$) or funds.

c) General case: Again, under the same conditions for seller of item i, we now consider that all users, $u \in U$, are bribable. The problem of finding the best bribing strategy is

$$\begin{cases} \text{maximize:} & \pi_{\sigma^i} \\ \text{subject to:} & \|\sigma^i\|_1 \leqslant J_i \\ & \sigma^i \in \mathcal{S}_i = \Xi_i \cup \bar{\Xi}_i \end{cases}$$

Next, we investigate when it is better to bribe a user $u \in U_i$ or a non-rater user $v \notin U_i$. For this, we consider the profit change rate, which are $\frac{\pi_{\sigma^i_u}}{\rho_u} = \delta$ and $\frac{\pi_{\sigma^i_v}}{\rho_u - r_i} = \gamma$, respectively. In the case, $c_u \geqslant c_v$ we always have $\delta \geqslant \gamma$. In the other case, $c_u < c_v$, we have $\gamma < \delta$ whenever either $\bar{c}_{U_i} < 1/|U_i|$ and $c_u < \alpha$, or $\bar{c}_{U_i} \geqslant 1/|U_i|$. Once again, the optimal strategy consists in ordering bribable users by decreasing reputation for each of the sets U_i and $U \backslash U_i$, and start allocating wealth first to the set of users that already rated item i and afterwards to the remaining users.

C. Properties of strategies and its profit in multipartite case

Now, we explore the profit of bribing for the multipartite reputation-based ranking system. To simplify the analysis, we assume that, when a user is bribed and changes his rating for an item, his reputation remains unchanged. This assumption is not very unrealistic since not only whenever the user has rated several items the change of its reputation is small when only one of his ratings change, but also because in real systems the re-computation of the reputations is often performed only from time to time. We assume that the users ratings and reputations are publicly available, but the network of users, that is, the partition into clusters is private for users' privacy reasons.

Proposition 6 (Bribing a user in a cluster that already rated the item)**.** *Suppose that $v \in U_i^{\mathcal{M}_s}$, for some cluster $s \in \{1, \dots, N\}$. If $c_v > \bar{c}_{U_i^{\mathcal{M}_s}}$, then any $\sigma_v \in \Xi_v$ is profitable.*

Proof. Following the same steps as in the proof of Proposition 4, replacing U_i by $U_i^{\mathcal{M}_s}$, we have that

$$\bar{\pi}_{\sigma_v^i} = \bar{J}_{\sigma_v^i} - \bar{J}_i = \rho_v \left(\frac{c_v}{\bar{c}_{U_i^{\mathcal{M}_s}}} - 1 \right) > 0. \qquad \square$$

Notice that, the previous result is the application of Proposition 4 to the cluster where the bribed user belongs.

Proposition 7 (Bribing a user in a cluster to rate a non-rated item in the cluster)**.** *Suppose that $v \in \mathcal{M}_s$, for a cluster $s \in \{1, \dots, N\}$, and consider an item, i, that was not rated by any member of the cluster, that is $i \notin I^{\mathcal{M}_s}$. In this case, any $\sigma_v \in \Xi_v$ is non-profitable.*

Proof. Recalling that $|U_i^{\mathcal{M}_s}| = 0$,

$$\begin{aligned}\bar{\pi}_{\sigma_v^i} = \sum_{m \in \mathcal{X}_i} \left| U_i^{\mathcal{M}_m} \right| r_i^{\mathcal{M}_m} + \left(\left| U_i^{\mathcal{M}_s} \right| + 1 \right) \frac{c_v \rho_v}{c_v} - \rho_v \\ - \sum_{m \in \mathcal{X}_i} \left| U_i^{\mathcal{M}_m} \right| r_i^{\mathcal{M}_m} = 0. \end{aligned} \qquad \square$$

Proposition 8 (Bribing a user in a cluster to rate an item that he did not rate before, but $i \in I^{\mathcal{M}_s}$)**.** *Suppose that we want to bribe a user that did not rate item i and the user belongs to a cluster where some user already rated item i, in other words, $v \in \mathcal{M}_s$, $v \notin U_i^{\mathcal{M}_s}$ and $i \in I^{\mathcal{M}_s}$. The strategy σ_v^i is profitable whenever one of the following holds:*

1) $c_v < \bar{c}_{U_i^{\mathcal{M}_s}}$ *and* $\rho_v < r_i^{\mathcal{M}_s}$
2) $c_v > \bar{c}_{U_i^{\mathcal{M}_s}}$ *and* $\rho_v > r_i^{\mathcal{M}_s}$.

Proof. Using an adaptation of (9), the profit of σ_v^i is:

$$\begin{aligned}\bar{\pi}_{\sigma_v^i} &= \left(\left| U_i^{\mathcal{M}_s} \right| + 1 \right) r_{\sigma_v^i}^{\mathcal{M}_s} - \rho_v - \left| U_i^{\mathcal{M}_s} \right| r_i^{\mathcal{M}_s} \\ &= \left(\alpha - |U_i^{\mathcal{M}_s}| c_v \right) \frac{r_i^{\mathcal{M}_s} - \rho_v}{\alpha + c_v},\end{aligned}$$

where $\alpha = \sum_{u \in U_i^{\mathcal{M}_s}} c_u$. To have a positive profit either 1) or 2) has to hold. $\square$

D. Optimal Strategies in multipartite scenario

Next, we study the optimal bribing strategies for the multipartite ranking system, as we did in Section III-B for the bipartite case. Again, we consider three scenarios: (i) bribing users that rated the item; (ii) bribing users that did not rate the item; (iii) bribing users from the set of all users. Finding the optimal strategies for these three cases can also be posed as solving problems of linear programming, therefore easy to solve. Moreover, in some cases, we can derive explicitly the optimal strategies and, hence, without the need of solving the associated linear optimization problem.

To model these problem we consider the following common set up. We assume that the seller of item i disposes of an initial wealth given by $\bar{J}_i$, recall (4). Moreover, we consider two reference costumers, u and v, with reputations s.t. $c_u > c_v$.

a) Bribing users that rated item i: Let us consider the case where the seller wants to bribe users that already rated item i, *i.e.* $u \in U_i$. We formulate this problem as

$$\begin{cases} \text{maximize:} & \bar{\pi}_{\sigma^i} \\ \text{subject to:} & \|\sigma^i\|_1 \leqslant \bar{J}_i \\ & \sigma^i \in \Xi_i \end{cases}$$

There are two cases to explore here: (i) both users belong to the same cluster; (ii) each user belongs to a different cluster.
(i) Suppose that $u, v \in \mathcal{M}_s$ are two users that already rated item i. As we show in Proposition 6, to have a positive profit $\bar{\pi}_{\sigma_u^i}$, when bribing user u, we need to have $c_u > \bar{c}_{U_i^{\mathcal{M}_s}}$. Hence, we do not consider strategies that bribe a user, v, s.t. $c_v < \bar{c}_{U_i^{\mathcal{M}_s}}$, because it would imply that the initial wealth, $\bar{J}_i$, would not increase.

Let $c_u > c_v > c_{U_i^{\mathcal{M}_s}}$, we compute the profit per unit of invested resources, $\bar{\pi}_{\sigma_u^i}/\rho_u - \bar{\pi}_{\sigma_v^i}/\rho_v = (c_u - c_v)/\bar{c}_{U_i^{\mathcal{M}_s}} > 0$. Thus, the profit per unit of invested wealth is larger for user u than for user v. Hence, as we obtained for the bipartite scenario, the optimal strategy is: to bribe users by decreasing reputation, investing all the available wealth until either the exhaustion of available profitable users ($c_u > \bar{c}_{U_i^{\mathcal{M}_s}}$) or the exhaustion of funds to bribe profitable users.

(ii) When each reference user belongs to distinct cluster, $u \in \mathcal{M}_s$, $v \in \mathcal{M}_t$ and $s \neq t$, we have that if $\left|U_i^{\mathcal{M}_s}\right| \geqslant \left|U_i^{\mathcal{M}_t}\right|$ then the profit per unit of invested wealth ($\bar{\pi}_{\sigma_u^i}/\rho_u$ versus $\bar{\pi}_{\sigma_v^i}/\rho_v$) is larger for user u. If $\left|U_i^{\mathcal{M}_s}\right| < \left|U_i^{\mathcal{M}_t}\right|$ then the profit per unit of invested wealth is larger for user u if both $\left|U_i^{\mathcal{M}_s}\right| > (c_u - c_v)^{-1}$ and $\left|U_i^{\mathcal{M}_t}\right| < \left(\left|U_i^{\mathcal{M}_s}\right| c_u - 1\right)/c_v$, otherwise it is larger for user v.

b) Bribing users that did not rate the item i: Under the same conditions for seller of item i, suppose that he wants to bribe users that did not rate item i, *i.e.* $u \notin U_i$. We formulate this problem as

$$\begin{cases} \text{maximize:} & \bar{\pi}_{\sigma^i} \\ \text{subject to:} & \|\sigma^i\|_1 \leqslant \bar{J}_i \\ & \sigma^i \in \bar{\Xi}_i \end{cases}$$

By recalling Proposition 7, we only need to explore the case where the sellers of item i want to bribe users belonging to clusters with users that already rated the item, clusters m s.t. $i \in I^{\mathcal{M}_m}$, otherwise the profit is zero. Let users $u, v \in \mathcal{M}_s$ and $u, v \notin U_i$ be s.t. $c_u > c_v$, and let $\alpha = \sum_{w \in U_i^{\mathcal{M}_s}} c_w$, $\gamma = \frac{\left|U_i^{\mathcal{M}_s}\right| c_u - \alpha}{c_u + \alpha}$ and $\delta = \frac{\left|U_i^{\mathcal{M}_s}\right| c_v - \alpha}{c_v + \alpha}$. The profit is given by Proposition 8, and we have for user u and v

$$\frac{\rho_u - r_i^{\mathcal{M}_s}}{c_u + \alpha}\left(\left|U_i^{\mathcal{M}_s}\right| c_u - \alpha\right) \text{ and } \frac{\rho_v - r_i^{\mathcal{M}_s}}{c_v + \alpha}\left(\left|U_i^{\mathcal{M}_s}\right| c_v - \alpha\right),$$

respectively. The difference of profits is $(\rho_u - r_i^{\mathcal{M}_s})\gamma - (\rho_v - r_i^{\mathcal{M}_s})\delta$, and hence for the same amount of spent wealth, $\bar{\pi}_{\sigma_u^i}/(\rho_u - r_i) > \bar{\pi}_{\sigma_v^i}/(\rho_v - r_i^{\mathcal{M}_s})$, because $\gamma > \delta$.

Again, the optimal strategy is to bribe users by decreasing order reputation, investing all the available wealth until either the exhaustion of profitable users ($c_u > \bar{c}_{U_i^{\mathcal{M}_s}}$) or funds.

In the case both users did not rate item i and they belong to different clusters, we cannot derive simple conditions and we need to solve the linear optimization problem for each instance of the problem.

c) General case: Again, under the same conditions for seller of item i, we now consider that all users, $u \in U$, can be bribed. In this case, the problem of finding the best bribing strategy may be written as

$$\begin{cases} \text{maximize:} & \bar{\pi}_{\sigma^i} \\ \text{subject to:} & \|\sigma^i\|_1 \leqslant \bar{J}_i \\ & \sigma^i \in \mathcal{S}_i = \Xi_i \cup \bar{\Xi}_i \end{cases}$$

Next, we investigate when it is better to bribe a user $u \in U_i^{\mathcal{M}_s}$ or a non-rater user $v \notin U_i^{\mathcal{M}_s}$. The result is the adaptation of the one for the general case in III-B. We consider the profit change rate, which are $\frac{\bar{\pi}_{\sigma_u^i}}{\rho_u} = \delta$ and $\frac{\bar{\pi}_{\sigma_v^i}}{\rho_u - \bar{r}_i} = \gamma$, respectively. In the case, $c_u \geqslant c_v$ we always have $\delta \geqslant \gamma$. In the other case, $c_u < c_v$, we have $\gamma < \delta$ whenever either $\bar{c}_{U_i^{\mathcal{M}_s}} < 1/|U_i^{\mathcal{M}_s}|$ and $c_u < \alpha$, or $\bar{c}_{U_i} \geqslant 1/|U_i^{\mathcal{M}_s}|$. Once again, the optimal strategy consists in ordering bribable users by decreasing reputation for each of the sets $U_i^{\mathcal{M}_s}$ and $U \backslash U_i \mathcal{M}_s$, and start allocating wealth first to the set of users that already rated item i and afterwards to the remaining users.

In the case that the reference users belong to different clusters, we cannot derive simple conditions and we need to solve the linear optimization problem for each instance of the problem.

E. Bipartite vs. multipartite networks

Here, we explore under which conditions it is more profitable to bribe a user in the scenario of the multipartite rating system versus the bipartite rating system. In the case the user already rated the item, it is easy to draw a condition we need to check to see whether it is more profitable to bribe in the bipartite ranking system or in the multipartite case.

Proposition 9. *Suppose that the seller of item i wants to bribe a user v that already rated the item,* i.e. $v \in U_i$. *Let the user v be in cluster $\mathcal{M}_s$, then the profit is larger in the bipartite case, $\bar{\pi}_{\sigma^i} < \pi_{\sigma_i}$, if and only if $\bar{c}_{(U_i \backslash U_i^{\mathcal{M}_s})} < \bar{c}_{U_i^{\mathcal{M}_s}}$, the average of the reputations in $(U_i \backslash U_i^{\mathcal{M}_s})$ and $U_i^{\mathcal{M}_s}$, respectively.*

Proof. By definition, $\bar{\pi}_{\sigma^i} < \pi_{\sigma_i}$ is the same as

$$\left(\frac{\left|U_i^{\mathcal{M}_s}\right| c_v}{\sum_{u \in U_i^{\mathcal{M}_s}} c_u} - 1\right)\rho_v < \left(\frac{|U_i|\, c_v}{\sum_{u \in U_i} c_u} - 1\right)\rho_v,$$

which is equivalent to

$$|U_i^{\mathcal{M}_s}| \sum_{u \in U_i} c_u < |U_i| \sum_{u \in U_i^{\mathcal{M}_s}} c_u.$$

Noticing that $U_i = U_i^{\mathcal{M}_s} \cup \left(U_i \backslash U_i^{\mathcal{M}_s}\right)$, we can rewrite it as

$$\left|U_i^{\mathcal{M}_s}\right| \left(\sum_{u \in U_i^{\mathcal{M}_s}} c_u + \sum_{u \in U_i \backslash U_i^{\mathcal{M}_s}} c_u \right) < \left(\left|U_i^{\mathcal{M}_s}\right| + \left|U_i \backslash U_i^{\mathcal{M}_s}\right| \right) \sum_{u \in U_i^{\mathcal{M}_s}} c_u,$$

Which, finally, is the same as $\bar{c}_{\left(U_i \backslash U_i^{\mathcal{M}_s}\right)} < \bar{c}_{U_i^{\mathcal{M}_s}}$. □

In other words, there exist cases where bribing a user under the multipartite scenario is more profitable than under the bipartite one, this is illustrated in the last two bars of the bar chart in Figure 3, Section IV-A, which correspond to bribe user u_5. However, as the clusters' partition is assumed to be unknown for the sellers, they cannot determine the users that verify the previous condition on the average of the reputations. Moreover, although this scenario is possible, it is not common for large clusters, where the reputation of the users tend to be similar and large, since the users have a similar rating pattern. But for small clusters, the average of its users' reputation may not exceed the average of the remaining users' reputation and, hence, the profit of bribing such a user is greater or equal than in the bipartite case. We see, in both the synthetic and real data examples in Section IV, that this rarely occurs.

Now, we compare the profit of bribing a user that did not rate the item i in the case the bribed user v belongs to a network where no users rated the item, $v \in \mathcal{M}_s$ and $i \notin I^{\mathcal{M}_s}$. In this case, bribing user v in the multipartite scenario yields a profit of zero, whereas in the bipartite the strategy can be profitable as we showed in Proposition 5.

In the case that the bribed user did not rate the item, but he belongs to a subnetwork where some user rated the item, we cannot draw simple conditions as in the previous cases. We need to check in each concrete scenario which is the most profitable case.

IV. Examples and simulations

In this section, we illustrate the main results of this paper with both synthetic and real data.

A. Synthetic data

Here, we explore the main results of this paper using synthetic generated data. We start by exploring Proposition 10.

Example 1. *Consider a scenario where $U_i = \{v\}$, $c_v = 1$, $c_w = 0.8$ and $R_{vi} = R_{wi} = 0.5$. In the case, consider strategy σ^i s.t. $\sigma_v^i = \sigma_w^i = 0.5$. We start by computing the profit of each elementary strategy. We have that $\pi_{\sigma_w^i} = (c_v - |U_i| c_w) \frac{r_i - \rho_w}{c_v + c_w} = 0$, and (after this strategy is applied) we have that $\pi_{\sigma_v^i} = (\frac{c_v}{c_v} |U_i| - 1)\rho_v = 0$. This yields a sum of the elementary strategies profit of 0. Whilst in the case of strategy σ^i, we have that $\pi_{\sigma^i} = \pi_{\sigma_v^i + \sigma_w^i} = \frac{c_v}{c_v + c_w} \pi_{\sigma_v^i} + \pi_{\sigma_w^i} + \frac{1}{c_v + c_w} \rho_v (c_v - c_w) = \frac{1}{18}$. The final reward is, hence, different for the two sequence of strategies.*

For the next examples, we consider 5 users, 2 items and 2 clusters of users. The ratings given by users to the items are presented in the first two rows of the users's columns in the first table of each example.

Example 2. *Consider $I = \{i, j\}$, $U = \{u_1, \ldots, u_5\}$ and two subnetworks $\mathcal{M}_1 = \{u_1, u_2, u_3\}$ and $\mathcal{M}_2 = \{u_4, u_5\}$. The users' reputations, the ratings given by users to items and the ranking of items for both the bipartite case and the multipartite case are summarized in Table I.*

	Users					Bipartite	Multipartite		
Items	u_1	u_2	u_3	u_4	u_5	r_x	$r_{x,\mathcal{M}_1}$	$r_{x,\mathcal{M}_2}$	$\bar{r}_x$
$x = i$	0.4	0.6	0.5	0.7	0.5	0.539	0.506	0.590	0.540
$x = j$	0.7	0.3	0.5	0.5	0.6	0.514	0.488	0.555	0.515
c_u	0.4	0.5	0.8	0.5	0.6				

TABLE I: Ratings given by users to items, users' reputations and items' rankings for both bipartite and multipartite cases.

Suppose the sellers of item i want to bribe a user in order to increase its ranking. In the first strategy, the sellers bribe user u_1, with $\sigma_{u_1}^i = 0.6$. In the second strategy, the sellers bribe user u_3, with $\sigma_{u_3}^i = 0.5$. The profits of each strategy, for both bipartite and multipartite cases, are represented in Table II. Using either the bipartite or the multipartite schemes, the

	Profit π		Ranking of o_1		
Strategy	Bipartite	Multipartite	r_i	$r_{i,\mathcal{M}_1}$	$\bar{r}_i$
$\sigma_{u_1}^i$	-0.171	-0.176	0.625	0.647	0.624
$\sigma_{u_3}^i$	0.214	0.206	0.682	0.741	0.706

TABLE II: Profits of bribing strategies $\sigma_{u_1}^i$ and $\sigma_{u_3}^i$ and new ranking, after applying the strategies, in both bipartite and multipartite cases.

strategy $\sigma_{u_1}^i$ is not profitable and strategy $\sigma_{u_3}^i$ is profitable. Further, the profit obtained in the bipartite case is larger that in

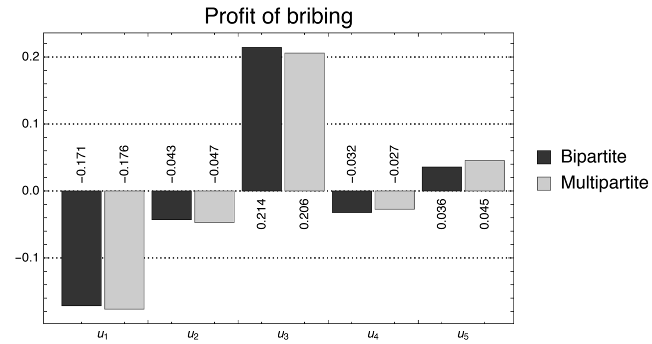

Fig. 3: Profit of i seller when bribing each user in the setup of Table I. The black and gray bars correspond to use the bipartite ranking and the multipartite ranking, respectively.

the multipartite cases for both strategies. Figure 3 depicts the profit of item i sellers when bribing each user, for both ranking systems.

Example 3. *Consider the same users, items and subnetworks as in Example 2, now with users' reputations, ratings given by users to items and ranking of items, for both the bipartite case and the multipartite case, summarized in Table III. Suppose*

	USERS					BIPARTITE	MULTIPARTITE		
ITEMS	u_1	u_2	u_3	u_4	u_5	r_x	$r_{x,\mathcal{M}_1}$	$r_{x,\mathcal{M}_2}$	$\bar{r}_x$
$x = i$	0.4	0.6	–	–	–	0.511	0.511	–	0.511
$x = j$	–	0.3	0.5	0.8	0.6	0.546	0.423	0.691	0.530
c_u	0.4	0.5	0.8	0.5	0.6				

TABLE III: Ratings given by users to items, users' reputations and items' rankings for both bipartite and multipartite case.

the sellers of item i want to bribe a user in order to get a larger ranking. In the first strategy, the sellers bribe user u_4, with $\sigma^i_{u_4} = 1$. In the second strategy, the sellers bribe user u_5, with $\sigma^i_{u_5} = 1$. The profit of each strategy for both bipartite and multipartite cases is represented in Table II. Using either the bipartite or the multipartite schemes, both strategies σ_{u_4} and σ_{u_5}

	PROFIT π		RANKING OF i		
STRATEGY	BIPARTITE	MULTIPARTITE	r_i	$r_{i,\mathcal{M}_2}$	$\bar{r}_i$
$\sigma^i_{u_4}$	0.035	0	0.686	1	0.707
$\sigma^i_{u_5}$	0.098	0	0.707	1	0.707

TABLE IV: Profits of bribing strategies $\sigma^i_{u_4}$ and $\sigma^i_{u_5}$ and new ranking, after applying the strategies, in both bipartite and multipartite cases.

are profitable for the bipartite case. However, both are not profitable for the multipartite case. The profit of bribing each user is depicted in Figure 4.

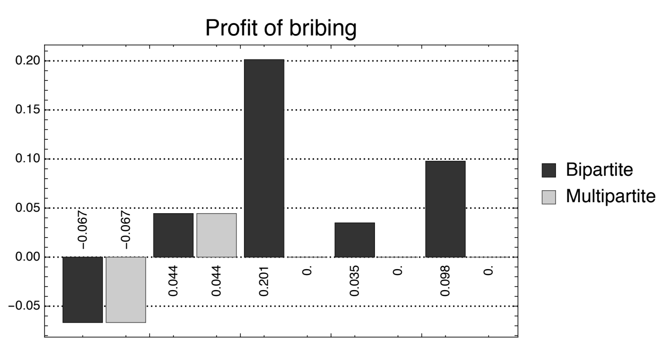

Fig. 4: Profit of i seller when bribing each user in the setup of Table III. The black and gray bars correspond to use the bipartite ranking and the multipartite ranking, respectively.

Note that in both Examples 2 and 3, the average and maximum of the profits for all elementary strategies are greater in the bipartite case, therefore in these examples the multipartite reputation-based ranking system is more robust to bribing, as expected.

B. Real data

Now, we explore bribing on the two studied reputation-based ranking systems with real data. We use the same dataset used in [10], the 5-core version of "Amazon Instant Video" dataset, [23]. This dataset consists of 5130 users, 1685 items, 37126 ratings and each user rated at least 5 items.

We simulate bribing strategies of the seller of the most rated item (with 455 ratings). We choose that item in order to have more data to explore, and the results would be similar for other items. Each strategy consists in bribing a set of users with a fixed size, 455, which is the number of raters of that item. Figure 5 depicts the simulation of different bribing strategies for both bipartite and multipartite scenarios, Figure 5 (a) and (b), respectively. We study the effect of the same four strategies in

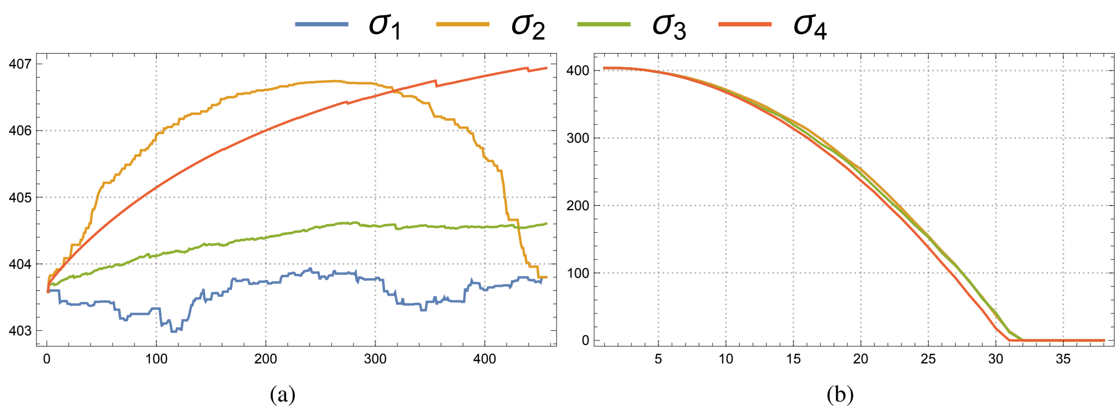

Fig. 5: Profit of bribing strategies of the most rated item sellers in both (a) bipartite ranking system ($\sigma_1, \sigma_2, \sigma_3$ and σ_4), and (b) multipartite ranking system (σ_2, σ_3 and σ_4). The number of users is only presented up to around 40 (not until the 455) because the wealth is already zero.

both bipartite and multipartite ranking systems, which are:

Strategy σ_1 – bribe users that rated the item, by a random order (only tested for the bipartite ranking system).

Strategy σ_2 – bribe users that rated the item, by decreasing reputation.

Strategy σ_3 – bribe users uniformly at random, from the set of all users.

Strategy σ_4 – bribe users by decreasing order of reputation from the set of all users.

The steps where the rewards is constant, in Figures 5 (a) and (b), represent choosing a users that already rated the item with the maximum allowed rating. We start by interpreting the results of the bribing strategies for the bipartite ranking system scenario, depicted in Figure 5 (a). After bribing the same users in strategies σ_1 and σ_2, both strategies yield the same profit, as stated in Proposition 1. Finally, the strategy σ_3 of Figure 5 (a) involves bribing users, from the set of all users, by decreasing reputation. As we intuitively expected, bribing users among the ones who rated the item and are more influential (with larger reputation) results in a faster increase of reward, whereas the random bribing among the item's raters has an expected profit close to zero, and does not increase wealth. The strategy σ_4 yields the largest profit, but involves bribing more users to achieve a profit larger than strategy σ_2. Nevertheless, in all strategies, the profit is positive.

Now, we explain the results for the multipartite ranking system scenario, depicted in Figure 5 (b). For the four strategies, the profit is almost always negative and the reward drops rapidly to zero. This illustrates the fact that the multipartite ranking system is much more robust to bribery than the bipartite ranking system, which meets the discussion in Section III-E.

Lastly, we consider applying strategy σ_2 to the bipartite case under the assumption that the reputations of users are fixed, and also without this restriction, the case when each time a user is bribed, not only the rankings of items are recomputed, but also the reputations of users are updated, as in [10]. The results of this experiment are depicted in Figure 6.

We can see, in Figure 6, that the reputations of the bribed users decrease and therefore the effect of their ratings has smaller impact, yielding a smaller profit than in the case where the reputations are fixed.

V. Conclusions and future work

We model bribing users of two reputation-based ranking systems that compute rankings as weighted average of users' ratings with their reputations. The first ranking system uses the main pool of users, whilst the second one clusters users by their similarities and presents, this way, a dedicated ranking of items for each cluster. We show which users, in both ranking systems, can be bribed with positive profit, and we show that the ranking system with clustering may decrease very much the number of profitable bribing strategies with the number of clusters. To illustrate our results, we explore using bribing strategies, for both ranking systems, in a real world dataset. In future work, we would like to study the interactions between bigger and smaller

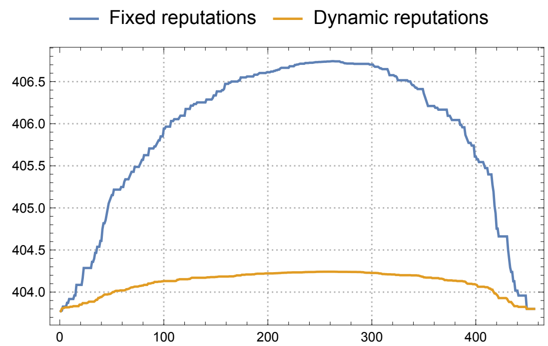

Fig. 6: Profit of bribing strategy σ_2 in the bipartite ranking system scenario for the case where the users' reputations are fixed versus the case the reputations are recomputed after each user being bribed.

players, as well as sellers bribing users to decrease a competitor item's ranking through a game theory model with the sellers as players. Another aspect to explore and incorporate in the bribery analysis is the dynamic behavior of the reputations that changed when the ratings changed, and therefore study new conditions to design profitable bribing strategies.

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