

- PEDRO BALTAZAR, CARLOS CALEIRO, SÉRGIO MARCELINO, *Conservativeness and decidability of fibred logics.*

SQIG - Instituto de Telecomunicações

Dep. Mathematics, IST - TU Lisbon, Portugal.

E-mail: `pedro.baltazar@ist.utl.pt, ccal@math.ist.utl.pt, sergiortm@gmail.com.`

The study of combined logics is certainly a key issue of the general theory of universal logic. Fibring is a very powerful and appealing mechanism for combining logics. As proposed by Dov Gabbay, fibring should combine \mathcal{L}_1 and \mathcal{L}_2 into a system which is the smallest logical system for the combined language which is a conservative extension of both \mathcal{L}_1 and \mathcal{L}_2 . Of course, a conservative extension of two given logics does not always exist, but in that case one should aim for being as conservative as possible.

We know that, given consequence relations for \mathcal{L}_1 and \mathcal{L}_2 , one just wants the smallest consequence relation over the combined language that extends the two. This corresponds precisely to putting together the axiomatizations of the two logics. However, even if some particular cases have been thoroughly studied and understood, e.g. fusion of modal logics, a general study of this syntactical construction is still lacking.

In this paper, we study general conditions for conservativity of fibred logics. To guarantee conservativity, it is certainly necessary that the two consequence systems are coincident when restricted to the common language. We show that this does not suffice and present extra conditions that do. In the particular case of unconstrained fibring - when the signatures are disjoint - we completely characterise the cases where it is conservative. Moreover, as a subproduct, we obtain that unconstrained fibring preserves decidability.

These results generalize what was known in the case of fusion of modal logics, covering also some well known open cases, like the fibring of classical and intuitionistic propositional logics.